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Valuation of Option Pricing with Meshless Radial Basis Functions Approximation

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Authors' contributions

This work was carried out in collaboration between both authors. Author MOD designed the study. Author JKO performed the numerical study and wrote the first draft of the manuscript. Authors MOD and JKO managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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ABSTRACT

This work focuses on valuation scheme of European and American options of single asset with meshless radial basis approximations. The prices are governed by Black – Scholes equations. The option price is approximated with three infinitely smooth positive definite radial basis functions (RBFs), namely, Gaussian (GA), Multiquadrics (MQ), Inverse Multiquadrics (IMQ). The RBFs were used for discretizing the space variables while Runge-Kutta method was used as a time-stepping marching method to integrate the resulting systems of differential equations. Numerical examples are shown to illustrate the strength of the method developed. The findings show that the RBFs has proven to be adaptable interpolation method because it does not depend on the locations of the approximation nodes which have overcome frequently evolving problems in computational finance such as slow convergent numerical solutions. Thus, the results allow concluding that the RBF-FD-GA and RBF-FD-MQ methods are well suited for modeling and analyzing Black and Scholes equation.

Keywords: European; American options; black-scholes equations; Radial Basis Functions (RBFs).

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1. INTRODUCTION

Black and Scholes established a mathematical model for calculating price of an option, which is now the basis of modern option pricing theory [1]. However, this model assumes that the price of the underlying asset S, satisfies under the risk neutral govern [2,3] the following stochastic differential equation

$$dS = rSdt + \sigma SdW$$

where r is the (constant) interest rate, σ is the (constant) volatility and W is a standard Wiener process. This model yields clear pricing formulae for some types of European options, comprising vanilla call and put. The exact closed-form solutions of the European options are available [2,3,4] but for American options closed-form solutions are not available, and numerical approximations are needed.

Some traditional approaches for solving the Black–Scholes equation are: the finite difference method (FDM) [5,6,7], penalty method [8,9], the finite element method (FEM) [10,11] the finite volume method (FVM) [10], the binomial and triniomial trees [11,12].

Radial basis function approximation has been proven to perform better than finite difference methods for option pricing problems in one and two dimensions [13,14].

In [15,16,17] global RBF method using Kansa's collocation approach for solving the one-dimensional Black–Scholes equation was discussed. The global RBF approximation leads to a dense linear system of equations which tends to be ill-conditioned when the shape parameter ε is small which caused a major setback for global RBF. Recently, a RBF-FD method has been proposed by Flyer and Fornberg [18]. The RBF-FD method generates a local RBF interpolant for expressing the function derivatives at a node as a linear combination of the function values on the present nodes in the neighborhood of the considered node [19,20]. Also, this RBF interpolants are used to generate the weights of a Finite Difference (FD) formula [21].

In this paper, the option price is approximated with three infinitely smooth positive definite radial basis functions-Finite Difference (RBF-FD), namely, Gaussian (GA), Multiquadrics (MQ), Inverse Multiquadrics (IMQ) which results in more accurate solutions and overcomes the difficulty of dense and ill-conditioned RBF matrices. The RBF-FD is used for discretizing the space variables while Runge-Kutta method was used as a time-stepping marching method to integrate the resulting systems of differential equations.

The rest of this paper is as follows. In Sect. 2, the option pricing is introduced briefly. Radial basis function approximation is introduced in Sect. 3. The numerical results and their interpretations are given in Sect. 4. We conclude this paper in Sect. 5.

2. METHODS

2.1 European Option: Governing Equation and Strike Condition

We first consider the values of the European options which can be solved by the following Black-Scholes equation:

$$\frac{\partial V(S,t)}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V(S,t)}{\partial S^2} + rS\frac{\partial V(S,t)}{\partial s} - rV = 0 \quad (2.1)$$

where r is the risk-free interest rate, σ is the volatility of stock price S, and $V(S,t)$ is the option value of the underlying asset (stock) at time t and stock price S.

For European call option, the terminal payoff valuation gives the strike condition:

$$V(S,t) = \max(S(T) - E, 0) \quad (2.2)$$

and, for European put option, the strike condition is as

$$V(S,t) = \max\,(E - S(T), 0) \quad (2.3)$$

Where S(T) is the terminal time and E is the strike price of the option. A simple transformation of $S = e^y$ changes the equation (1.1) and the initial condition (1.2) and (1.3) to:

$$\frac{\partial U}{\partial t} + \frac{1}{2}\sigma^2\frac{\partial^2 U}{\partial y^2} + \left(r - \frac{1}{2}\sigma^2\right)\frac{\partial U}{\partial y} - rU = 0 \quad (2.4)$$

For European call option, the initial condition:

$$U\left(\underline{y}, T\right) = \max(e^{\underline{y}} - E, 0) \quad (2.5)$$

and, for European put option, the initial condition is

$$U\left(\underline{y}, T\right) = \max\,(E - e^{\underline{y}}, 0) \quad (2.6)$$

European options are subjected to the following boundary conditions:

$$V(0,t) = Xe^{-r(T-t)}, \quad V(S,t) \to \infty as S \to \infty \quad for put option$$
$$V(0,t) = 0 \quad V(S,t) \to \infty as S \to \infty \quad for call option \tag{2.7}$$

The exact solution of Black-Scholes equation (2.1) subject to the strike condition (2.2) and (2.3) and the boundary conditions (2.7 can be obtained by:

$$V(S,t) = Xe^{-r(T-t)}N(-d_2) - SN(-d_1) \quad for put option$$
$$V(S,t) = SN(d_1) - Xe^{-r(T-t)}N(d_2) \quad for call option \tag{2.8}$$

where N(.) is the cumulative normal distribution

$$d_1 = \frac{\log\left(\frac{S}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \tag{2.9}$$

$$d_2 = \frac{\log\left(\frac{S}{X}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \tag{2.10}$$

2.2 American Option

The valuation of the American put options can be considered as a free boundary value problem because it can be exercised any time before maturity with optimal exercise stock value $S = B(t)$ and does not have an exact closed-form solution due to the unknown free boundary $B(t)$ (Wilmott [3]; Hon and Mao [4]). American options allow for early exercises at any time $t \in [0, T]$ with optimal exercise.

Recently, optimal exercise for American options requires the following boundary conditions:

$$V(S,t) = \max\{V(S,T), V(S,t)\} \tag{2.11}$$

Where, the terminal payoff $V(S,T)$ is given by the initial condition (2.2) and (2.3) for both put and call options. The movement of the unknown free boundary B(t) differentiates pricing of the American options from the European options which enforces additional restriction at any time t on the solution that its value must be at least $V(S,T)$ (Khabir [22] and Wu and Hon [23]).

3. RESULTS

Given N distinct data points (centers), $\underline{y}_k$, we construct an interpolant of the form

$$U\left(\underline{y},T\right) = \sum_{k=1}^{N} \lambda_k(t)\phi\left(\left\|\underline{y} - \underline{y}_k\right\|\right) + p_l\left(\underline{y}\right) \tag{3.1}$$

With the constraints

$$\sum_{k=1}^{N} \lambda_k(t) p_l\left(\underline{y}_k\right) = 0 \tag{3.2}$$

was employed over all the N node locations in the domain of interest. Here, $\lambda_k(t)$s are unknown coefficients depending on t, ||·|| denotes Euclidean distance, $\phi\left(\underline{y}\right)$ is some radial function and $p_l\left(\underline{y}\right)$ are all polynomials up to degree l in the dimension of the problem considered. However, these additional constraints make sure that the radial basis function reproduces polynomials up to degree l and it ensures that the radial basis function expansion does not blow up [24].

The primary choice in the implementation is what functions $\phi\left(\underline{y}\right)$ to use. In this paper, we will use three of the most common choices listed in Table 1for our analysis. A clear difference is made here between three different categories: the infinitely smooth, the piecewise smooth and the compactly supported RBFs. Radial basis functions of the first type are $C^{\infty}(0,\infty)$ and can provide spectral accuracy, while the piecewise smooth RBFs give algebraic convergence for interpolation [25].

The best choice for the shape parameter "c" in the infinitely smooth radial basis functions has been subject to extensive study. Once a smooth interpolant $U(y,T)$ to the scattered data has been found, it becomes possible to differentiate it and thereby obtain accurate approximations to partial derivatives. The use of such approximations for the numerical solution of PDEs was pioneered around 1990 by E. Kansa for elliptic, parabolic, and certain hyperbolic problems.

In (2.4), we choose N collocation points, y_k, and yield the following systems of N equations:

$$\frac{\partial U(y_i,t)}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 U(y_i,t)}{\partial y^2} + \left(r - \frac{1}{2}\sigma^2\right)\frac{\partial U(y_i,t)}{\partial y} - rU(y_i,t) = 0 \tag{3.3}$$

Where,

$$\frac{\partial U(y_i,t)}{\partial t} = \sum_{k=1}^{N} \frac{d\lambda_k(t)}{dt} \phi(y_i, y_k) \tag{3.4}$$

$$\frac{\partial U(y_i,t)}{\partial y} = \sum_{k=1}^{N} \lambda_k(t) \frac{\partial \phi(y_i, y_k)}{\partial y} \tag{3.5}$$

$$\frac{\partial^2 U(y_i,t)}{\partial y^2} = \sum_{k=1}^{N} \lambda_k(t) \frac{\partial^2 \phi(y_i,y_k)}{\partial y^2} \tag{3.6}$$

Taking the infinitely smooth radial basis function:

For Gaussian

$$(\frac{\partial \phi(y_i,y_k)}{\partial y} = -2c^2(y_i - y_k)e^{-c^2(y_i-y_k)^2} \tag{3.7}$$

$$\frac{\partial^2 \phi(y_i,y_k)}{\partial y^2} = -2c^2 e^{-c^2(y_i-y_k)^2} + 4c^4(y_i - y_k)^2 e^{-c^2(y_i-y_k)^2} \tag{3.8}$$

For Multiquadrics (MQ):

$$\frac{\partial \phi(y_i,y_k)}{\partial y} = \frac{(y_i-y_k)}{\sqrt{(y_i-y_k)^2+c^2}} \tag{3.9}$$

$$\frac{\partial^2 \phi(y_i,y_k)}{\partial y^2} = \frac{1}{\sqrt{(y_i-y_k)^2+c^2}} - \frac{(y_i-y_k)^2}{((y_i-y_k)^2+c^2)^{\frac{3}{2}}} \tag{3.10}$$

For Inverse Multiquadrics (IMQ):

$$\frac{\partial \phi(y_i,y_k)}{\partial y} = -\frac{(y_i-y_k)}{((y_i-y_k)^2+c^2)^{\frac{3}{2}}} \tag{3.11}$$

$$\frac{\partial^2 \phi(y_i,y_k)}{\partial y^2} = -\frac{(y_i-y_k)}{((y_i-y_k)^2+c^2)^{\frac{3}{2}}} + \frac{3(y_i-y_k)^2}{((y_i-y_k)^2+c^2)^{\frac{5}{2}}} \tag{3.12}$$

For Inverse Quadrics (IQ):

$$\frac{\partial \phi(y_i,y_k)}{\partial y} = \frac{2c^2(y_i-y_k)}{(1+c^2(y_i-y_k)^2)^2} \tag{3.13}$$

$$\frac{\partial^2 \phi(y_i,y_k)}{\partial y^2} = \frac{2c^2(y_i-y_k)}{(1+c^2(y_i-y_k)^2)^2} - \frac{8c^4(y_i-y_k)^2}{(1+c^2(y_i-y_k)^2)^2} \tag{3.14}$$

In matrix form, (3.3) becomes

$$\mathbf{\Phi}\dot{\alpha} + \frac{1}{2}\sigma^2 \mathbf{\Phi}_{yy}\alpha + \left(r - \frac{1}{2}\sigma^2\right)\mathbf{\Phi}_y \alpha - r\mathbf{\Phi}\alpha = 0 \tag{3.15}$$

This results in the system of linear homogenous ODEs for the coefficients λ_k (collected in the vector α), while the $\mathbf{\Phi}, \mathbf{\Phi}_y$ and $\mathbf{\Phi}_{yy}$ are the matrices with entries $\phi_k(y_k)$, $\frac{\partial \phi(y_i,y_k)}{\partial y}$ and $\frac{\partial^2 \phi(y_i,y_k)}{\partial y^2}$ respectively.

Applying RBF-FD plus linear polynomial terms, the RBF-FD weights $w_k, k = 1,2,\ldots,n$ in 2-D case are found by solving

$$\left[\begin{array}{ccc|ccc} \phi\left(\left\|\underline{y}_1 - \underline{y}_1\right\|\right) & \cdots & \phi\left(\left\|\underline{y}_1 - \underline{y}_n\right\|\right) & 1 & x_1 & y_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \phi\left(\left\|\underline{y}_n - \underline{y}_1\right\|\right) & \cdots & \phi\left(\left\|\underline{y}_n - \underline{y}_n\right\|\right) & 1 & x_n & y_n \\ \hline 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & \cdots & y_n & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \hline w_{n+1} \\ w_{n+2} \\ w_{n+3} \end{bmatrix} = \begin{bmatrix} L\phi\left(\left\|\underline{y} - \underline{y}_1\right\|\right)\big|_{\underline{y}=\underline{y}_c} \\ \vdots \\ L\phi\left(\left\|\underline{y} - \underline{y}_n\right\|\right)\big|_{\underline{y}=\underline{y}_c} \\ \hline L\,1\big|_{\underline{y}=\underline{y}_c} \\ Lx\big|_{\underline{y}=\underline{y}_c} \\ Ly\big|_{\underline{y}=\underline{y}_c} \end{bmatrix} \tag{3.16}$$

The derivation of the above linear system for differentiation weights is given [26], see section (5.1.4). The weights w_{n+1} to w_{n+3} are disregarded after the matrix is inverted. Solving (3.16) gives one row of

differentiation matrix (DM) that includes the weights for approximating Lat $\underline{y}_c$. A preprocessing cost of $O(n^3N)$ is incurred when it is repeated N times over all nodes in the domain of interest.
In order to resolve the time component, we use an Explicit RK4 method, i.e.,

$$\dot{\alpha} = \mathbf{\Phi}^{-1}\left[\tfrac{1}{2}\sigma^2\mathbf{\Phi}_{yy}\alpha + \left(r - \tfrac{1}{2}\sigma^2\right)\mathbf{\Phi}_y\alpha - r\mathbf{\Phi}\alpha\right] \equiv L\alpha \tag{3.17}$$

Where

$$L = rI - \frac{1}{2}\sigma^2\mathbf{\Phi}^{-1}\mathbf{\Phi}_{yy} - \left(r - \frac{1}{2}\sigma^2\right)\mathbf{\Phi}^{-1}\mathbf{\Phi}_y$$

The explicit first order backward difference time integration scheme (BD

$$\alpha_n = \alpha_{n-1} - \Delta t L\alpha_{n-1} \tag{3.18}$$

Let,

$$W_1 = -\Delta t L\alpha_{n-1} W_2 = -\Delta t L(\alpha_{n-1} + 0.5W_1) W_3 = -\Delta t L(\alpha_{n-1} + 0.5W_2) W_4 = -\Delta t L(\alpha_{n-1} + W_3)$$

$$\alpha_n = \alpha_{n-1} + \frac{W_1 + 2W_2 + 2W_3 + W_4}{6}$$

Table 1.Examples of commonly used radial basis functions

Name	$\boldsymbol{\phi(r)}$	**Notes**
	Infinitely smooth	
Gaussian (GA)	$e^{-(\varepsilon r)^2}$	$\varepsilon \in \mathbb{R}$
Multiquadrics (MQ)	$\sqrt{1+(\varepsilon r)^2}$	$\varepsilon \in \mathbb{R}$
Inverse Multiquadrics (IMQ)	$\frac{1}{\sqrt{1+(\varepsilon r)^2}}$	$\varepsilon \in \mathbb{R}$
Inverse Quadratic (IQ)	$\frac{1}{1+(\varepsilon r)^2}$	$\varepsilon \in \mathbb{R}$
Generalized Multiquadric	$(1+(\varepsilon r)^2)^\beta$	$\varepsilon \in \mathbb{R}, \beta \in \mathbb{R}\backslash\mathbb{N}_0$
Laguerre–Gaussian	$(2-(\varepsilon r)^2)e^{-(\varepsilon r)^2}$	$\varepsilon \in \mathbb{R}^2$, oscillatory
Poisson	$\sqrt{\frac{\pi}{2}}\frac{\sin(\varepsilon r)}{\varepsilon r}$	$\varepsilon \in \mathbb{R}^3$, oscillatory
	Piecewise smooth	
Radial power	r^β	$0 < \beta \notin 2\mathbb{N}$
Thin plate spline	$r^{2\beta}\log r$	$\beta \in \mathbb{N}$
	Compactly supported	
Wendland	$(1-r)_+^6(35r^2+18r+3)$	Ex., C^4 in $\mathbb{R}^3$
Gneiting	$(1-r)_+^5(1+5r-27r^2)$	Ex., C^2 in $\mathbb{R}^2$

4. DISCUSSION

4.1 Test Problem 1

The type of basis functions we selected are infinitely smooth and less sensitive to the choice of the shape parameter than, e.g., the Gaussian RBF. We use the following set of parameters for European put option with X = 20, T = 0.25, r = 0.05, sigma = 0.2, n= 101, m=60, Smin=1, Smax=30 and the option prices are computed for stock prices S=[2,4,6,8,10,12,14,16,18,20].

Table 2 shows the prices, the approximations errors, the RMSE and the CPU time of the pricing of European put options. The option prices are computed by the RBF-FD-GA, RBF-FD-MQ and RBF-FD-IMQ interpolation respectively. The approximations errors, the RMSE are defined in terms of a monetary unit ($ US dollar). The CPU time is reported in seconds. For European put options the benchmark is analytical solution obtained from Black and Scholes equations and compared with what result obtained by method proposed. From Table 2 one can observe that we got reasonable accurate result using the radial basis function-finite difference method, in the sense that it is much closer to the analytical solution.

Table 2. Comparison of the values of the European Put Options

Stock Price	Exact Solution	RBF-FD			Approximation error		
		GA(c = 11.5306)	MQ(c= 0.0972)	IMQ(c= 0.0925)	GA	MQ	IMQ
2	17.7516	17.7516	17.7516	17.5681	-4.24E-05	1.01E-06	1.83E-01
4	15.7516	15.7516	15.7516	15.5865	-3.78E-05	4.58E-07	1.65E-01
6	13.7516	13.7516	13.7516	13.6072	-7.91E-06	1.63E-06	1.44E-01
8	11.7516	11.7516	11.7516	11.6276	-2.52E-06	2.69E-06	1.24E-01
10	9.7516	9.7514	9.7516	9.6471	1.47E-04	3.45E-06	1.04E-01
12	7.7516	7.7515	7.7516	7.6656	5.67E-05	3.93E-06	8.60E-02
14	5.7517	5.7518	5.7517	5.6828	-1.08E-04	2.98E-06	6.89E-02
16	3.7628	3.7628	3.7629	3.6988	6.31E-05	1.01E-05	6.41E-02
18	1.9311	1.9306	1.9309	1.7032	4.36E-04	1.37E-04	2.28E-01
20	0.6746	0.6735	0.6740	-0.0968	1.05E-03	5.32E-04	7.71E-01
RMSE					3.65E-04	1.74E-04	2.78E-01
Time					0.31	0.51	0.57

Table 3. Comparison of the values of the European Call Options

Stock Price	Exact Solution	RBF-FD			Approximation error		
		GA (c = 16.5178)	MQ (c= 0.1638)	IMQ (c= 0.1113)	GA	MQ	IMQ
2	1.26E-16	1.18E-16	-7.44E-07	-0.0432	7.32E-18	7.44E-07	4.32E-02
4	1.07E-06	1.06E-06	1.59E-06	-0.0592	6.58E-09	5.25E-07	5.92E-02
6	0.0038	0.0038	0.0038	-0.0796	5.60E-07	3.88E-07	8.34E-02
8	0.1493	0.1494	0.1494	-0.1105	-7.51E-05	7.49E-05	2.60E-01
10	0.9163	0.9165	0.9165	0.0449	-2.54E-04	2.54E-04	8.71E-01
12	2.4144	2.4146	2.4146	2.0191	-2.26E-04	2.26E-04	3.95E-01
14	4.2795	4.2796	4.2796	3.9442	-9.92E-05	1.00E-04	3.35E-01
16	6.2523	6.2523	6.2523	5.8717	-2.76E-05	2.94E-05	3.81E-01
18	8.2477	8.2477	8.2477	7.7958	-3.97E-06	6.52E-06	4.52E-01
20	10.2470	10.2470	10.2470	9.7219	1.84E-06	1.53E-06	5.25E-01
RMSE					1.15E-04	1.15E-04	4.16E-01
Time					0.48	0.37	0.41

Table 4. Comparison of the values of the American Put Options

Stock Price	Binomial (n=1000)	RBF-FD			Approximation error		
		GA (c = 16.2455)	MQ (c= 0.1611)	IMQ (c= 0.1095)	GA	MQ	IMQ
8	2.0000	1.9972	1.9972	1.9981	2.77E-03	2.78E-03	1.88E-03
9	1.0305	1.0277	1.0277	0.9915	2.72E-03	2.73E-03	3.90E-02
10	0.4025	0.4021	0.4021	0.0305	3.28E-04	3.17E-04	3.72E-01
11	0.1207	0.1206	0.1206	-0.0214	6.32E-05	4.70E-05	1.42E-01
12	0.0282	0.0282	0.0282	-0.0162	-4.35E-05	5.15E-05	4.44E-02
13	0.0053	0.0053	0.0053	-0.0147	-3.12E-05	3.53E-05	2.00E-02
14	0.0008	0.0008	0.0008	-0.0141	-1.24E-05	1.43E-05	1.49E-02
15	0.0001	0.0001	0.0001	-0.0129	-2.90E-06	2.56E-06	1.30E-02
16	1.32E-05	1.37E-05	1.20E-05	-0.0110	-4.12E-07	1.28E-06	1.10E-02
17	1.43E-06	1.50E-06	2.39E-08	-0.0098	-7.22E-08	1.41E-06	9.76E-03
18	1.44E-07	1.87E-07	-7.87E-07	-0.0096	-4.30E-08	9.31E-07	9.64E-03
19	1.35E-08	6.80E-09	-1.22E-06	-0.0093	6.74E-09	1.24E-06	9.27E-03
20	1.21E-09	2.12E-09	-1.33E-06	-0.0086	-9.04E-10	1.33E-06	8.61E-03
RMSE					1.08E-03	1.09E-03	1.12E-01
TIME					70.46	59.02	59.14

Table 5. Comparison of the values of the American Put Options

Stock Price	Binomial (n=1000)	RBF-FD			Approximation error		
		GA (c = 16.0476)	MQ (c= 0.1591)	IMQ (c= 0.1095)	GA	MQ	IMQ
4	0.0646	0.0647	0.0647	1.9981	-8.18E-05	7.52E-05	1.93E+00
5	0.2279	0.2281	0.2281	0.9915	-1.79E-04	1.73E-04	7.64E-01
6	0.5416	0.5417	0.5417	0.0305	-6.00E-05	5.47E-05	5.11E-01
7	1.0134	1.0134	1.0134	-0.0214	-1.61E-05	1.15E-05	1.03E+00
8	1.6256	1.6258	1.6257	-0.0001	-1.95E-04	1.91E-04	1.63E+00
9	2.3508	2.3508	2.3508	0.9987	-1.21E-05	8.20E-06	1.35E+00
10	3.1602	3.1603	3.1602	1.9979	-4.22E-05	3.79E-05	1.16E+00
11	4.0301	4.0302	4.0302	2.9970	-6.11E-05	5.83E-05	1.03E+00
12	4.9421	4.9422	4.9422	3.9961	-1.76E-04	1.75E-04	9.46E-01
13	5.8828	5.8831	5.8831	4.9952	-2.95E-04	2.95E-04	8.88E-01
14	6.8433	6.8435	6.8435	5.9943	-1.92E-04	1.92E-04	8.49E-01

Stock Price	**Binomial (n=1000)**	**RBF-FD**			**Approximation error**		
		GA (c = 16.0476)	**MQ (c= 0.1591)**	**IMQ (c= 0.1095)**	**GA**	**MQ**	**IMQ**
15	7.8168	7.8169	7.8169	6.9934	-7.12E-05	6.59E-05	8.23E-01
16	8.7989	8.7990	8.7990	7.9925	-1.11E-04	9.89E-05	8.06E-01
17	9.7870	9.7870	9.7870	8.9916	-5.01E-05	4.82E-05	7.95E-01
18	10.7788	10.7789	10.7789	9.9907	-5.55E-05	7.07E-05	7.88E-01
19	11.7733	11.7734	11.7734	10.9898	-9.66E-05	8.04E-05	7.84E-01
20	12.7696	12.7697	12.7697	11.9889	-7.54E-05	6.42E-05	7.81E-01
RMSE		1.2808E-04	1.2514E-04	1.0503E+00	1.28E-04	1.25E-04	1.05E+00
TIME					77.17	77.30	59.14

4.2 Test Problem 2

The type of basis functions we selected are infinitely smooth and less sensitive to the choice of the shape parameter than, e.g., the Gaussian RBF. We use the following set of parameters for European call option with X = 10.0, T = 0.25, r = 0.1, sigma = 0.4, n= 101, m=200, Smin=1, Smax=60 and the option prices are computed for stock prices S=[2,4,6 ,8,10,12,14,16,18,20].

Table 3 shows the prices, the approximations errors, the RMSE and the CPU time of the pricing of European call options. The option prices are computed by the RBF-FD-GA, RBF-FD-MQ and RBF-FD-IMQ interpolation respectively. The approximations errors, the RMSE are defined in terms of a monetary unit ($ US dollar). The CPU time is reported in seconds. For European call options the benchmark is analytical solution obtained from Black and Scholes equations and compared with what result obtained by method proposed. From Table 3 one can observe that we got reasonable accurate result using the radial basis function-finite difference method, in the sense that it is much closer to the analytical solution.

4.3 Test Problem 3

The type of basic functions we selected are infinitely smooth and less sensitive to the choice of the shape parameter than, e.g., the Gaussian RBF. We use the following set of parameters for American put option with $X = 10.0$, $T = 0.25$, $r = 0.1$, $sigma = 0.25$, $n = 150$, $m = 101$, $Smin = 1$, $Smax = e^6$ and the option prices are computed for stock prices $S = [8,9,10,11,12,13,14,15,16,17,18,19,20]$.

Table 4 shows the prices, the approximations errors, the RMSE and the CPU time of the pricing of American put options. The option prices are computed by the RBF-FD-GA, RBF-FD-MQ and RBF-FD-IMQ interpolation respectively. The approximations errors, the RMSE are defined in terms of a monetary unit ($ US dollar). The CPU time is reported in seconds. For American put options the benchmark is analytical solution obtained from Binomial tree and compared with what result obtained by method proposed. From Table 4 one can observe that we got reasonable accurate result using the radial basis function-finite difference method, in the sense that it is much closer to the analytical solution.

4.4 Test Problem 4

The type of basis functions we selected are infinitely smooth and less sensitive to the choice of the shape parameter than, e.g., the Gaussian RBF. We use the following set of parameters for American call option with $X = 8.0$, $T = 1$, $r = 0.1$, $sigma = 0.4$, $n = 150$, $m = 20$, $Smin = 0.4$, $Smax = 150$ and the option prices are computed for stock prices $S = [4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20]$.

Table 5 shows the prices, the approximations errors, the RMSE and the CPU time of the pricing of American call options. The option prices are computed by the RBF-FD-GA, RBF-FD-MQ and RBF-FD-IMQ interpolation respectively. The approximations errors, the RMSE are defined in terms of a monetary unit ($ US dollar). The CPU time is reported in seconds. For American call options the benchmark is analytical solution obtained from Binomial tree and compared with what result obtained by method proposed. From Table 5 one can observe that we got reasonable accurate result using the radial basis function-finite difference method, in the sense that it is much closer to the analytical solution.

5. CONCLUSION

In this work, we have discussed the valuation scheme of European and American options of single asset with meshless radial basis approximations. The prices are governed by Black – Scholes equations. The option price is approximated with three infinitely smooth positive definite radial basis functions (RBFs), namely, Gaussian (GA), Multiquadrics (MQ), InverseMultiquadrics (IMQ). The RBFs were used for discretizating the space variables while Runge-Kutta method was used as a time-stepping marching method to integrate the resulting systems of differential equations. The findings show that the RBFs has proven to be adaptable interpolation method because it does not depend on the locations of the approximation nodes which have overcome frequently evolving problems in computational finance such as slow convergent numerical solutions. The results show that the RBF-FD-GA and RBF-FD-MQ achieve more accurate and faster option prices are in good agreement with analytical solutions than RBF-FD-IMQ. Thus, the results allow concluding that the RBF-FD-GA and RBF-FD-MQ methods are well suited for modeling and analyzing Black and Scholes equation.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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